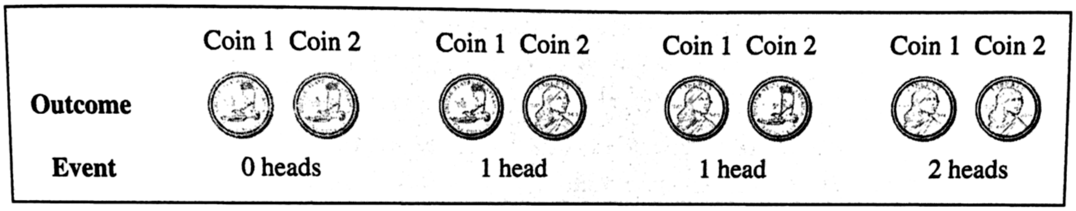

FIGURE 7.1 The four possible outcomes for a toss of two coins. The two middle outcomes both represent the same event of 1 head.

Definitions

Outcomes are the most basic possible results of observations or experiments. For example, if you toss two coins, one possible outcome is HT and another possible outcome is TH.

An **event** consists of one or more outcomes that share a property of interest. For example, if you toss two coins and count the number of heads, the outcomes HT and TH both represent the same *event* of 1 head (and 1 tail).

EXAMPLE 1 Family Outcomes and Events

Consider families with two children. List all the possible *outcomes* for the birth order of boys and girls. If we are only interested in the total number of boys in the families, what are the possible *events*?

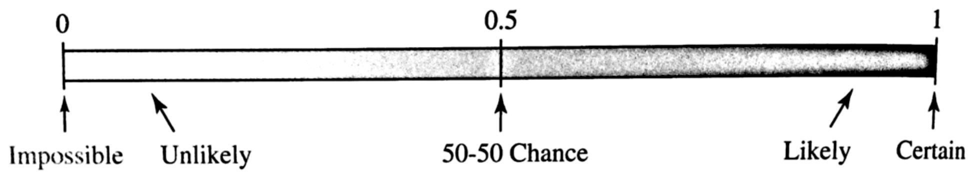

FIGURE 7.2 The scale shows various degrees of certainty as expressed by probabilities.

Expressing Probability

The probability of an event, expressed as $P(\text{event})$, is always between 0 and 1 (inclusive). A probability of 0 means the event is impossible and a probability of 1 means the event is certain.

Note on rounding: If possible, express a probability as an *exact* fraction or decimal; otherwise, round the result, usually to three significant digits, as in 0.00457.

Theoretical Method for Equally Likely Outcomes

Step 1. Count the total number of possible outcomes.

Step 2. Among all the possible outcomes, count the number of ways the event of interest, A, can occur.

Step 3. Determine the probability, $P(A)$, from

$$P(A) = \frac{\text{number of ways } A \text{ can occur}}{\text{total number of possible outcomes}}$$

EXAMPLE 2 Coins and Dice

Apply the theoretical method to find the probability of:

a. exactly one head when you toss two coins.

b. getting a 3 when you roll a 6-sided die.

EXAMPLE 3 **Playing Card Probabilities**

Figure 7.4 shows the 52 playing cards in a standard deck. There are four *suits*: hearts, spades, diamonds, and clubs. Each suit has cards labeled with the numbers 2 through 10, plus a jack, queen, king, and ace (for a total of 13 cards in each suit). Notice that hearts and diamonds are red, while spades and clubs are black. If you draw one card at random from a standard deck, what is the probability that it is a spade?

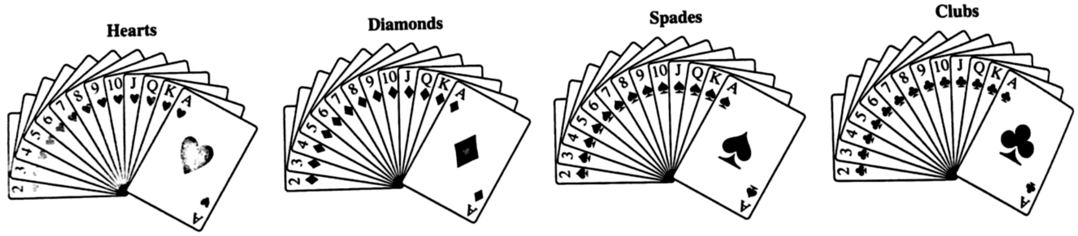

FIGURE 7.4 Playing cards in a standard 52-card deck.

EXAMPLE 4 **Two Girls and a Boy**

What is the probability that a randomly selected family with three children has two girls and one boy? Assume boys and girls are equally likely.

boy is 3/8, or 0.375. ▶ **Now try Exercises 23–24.**

EXAMPLE 5 **Birth Month Probability**

You select a person at random from a large group attending a conference. What is the probability that the person has a birthday in July? Assume that there are 365 days in a year and births are equally likely to occur on any day of the year.

Relative Frequency Method

Step 1. Repeat or observe a process many times and count the number of times the event of interest, A, occurs.

Step 2. Estimate $P(A)$ using this formula:

$$P(A) = \frac{\text{number of times } A \text{ occurred}}{\text{total number of observations}}$$

EXAMPLE 6 **500-Year Flood**

Geological records indicate that a river has crested above a particular high flood level 4 times in the past 2000 years. Using the relative frequency method, what is the probability that the river will crest above this flood level next year?

single year. ▲

▶ **Now try Exercises 29–30.**

EXAMPLE 7 **Fair Coin Test**

Suppose you toss two coins 100 times and observe the following results:

- 0 heads occurs 22 times.
- 1 head occurs 51 times.
- 2 heads occurs 27 times.

Compare the relative frequency probabilities to the theoretical probabilities. Do you have reason to suspect that the coins are unfair?

Three Approaches to Finding Probability

A **theoretical probability** is based on the assumption that all outcomes are equally likely. It is calculated by dividing the number of ways an event can occur by the total number of possible outcomes.

A **relative frequency probability** is based on observations or experiments. It is the relative frequency of the event of interest.

A **subjective probability** is an estimate based on experience or intuition.

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EXAMPLE 8 **Which Method?**

Identify the method that resulted in the following statements.

a. I'm certain that you'll be happy with this car.
b. Based on housing data, the chance that someone will move to a new residence during the coming year is about 1 in 8.
c. The probability of rolling a 7 with a 12-sided die is 1/12.

Probability of an Event *Not* Occurring

If the probability of an event A is $P(A)$, we write the probability that event A does *not* occur as $P(\text{not } A)$. Because an event either does or does not occur, the sum of these probabilities is $P(A) + P(\text{not } A) = 1$. Therefore, the probability that event A does *not* occur is

$$P(\text{not } A) = 1 - P(A)$$

EXAMPLE 9 **Not Two Girls**

What is the probability that a randomly chosen family with three children does *not* hav two girls and one boy? Assume boys and girls are equally likely.

Making a Probability Distribution

A **probability distribution** represents the probabilities of all possible events of interest. To make a table of a probability distribution:

Step 1. List all possible *outcomes*.

Step 2. Identify outcomes that represent the same *event*. Find the probability of each event.

Step 3. Make a table or figure that displays all the probabilities. The sum of all the probabilities must be 1.

EXAMPLE 10 Tossing Three Coins

Make a probability distribution for the number of heads that occur when three coins are tossed simultaneously.

EXAMPLE 11 Two Dice Distribution

Make a probability distribution for the sum of the dice when two fair dice are rolled together. What is the most probable sum?

Definition

The **odds for** an event A are given by

$$\text{odds for event } A = \frac{P(A)}{P(\text{not } A)}$$

The **odds against** an event A are given by

$$\text{odds against event } A = \frac{P(\text{not } A)}{P(A)}$$

Note: In gambling, the term *odds on* generally means "odds against."

EXAMPLE 12 Two-Coin Odds

What are the odds for getting two heads when tossing two coins together? What are the odds against it?

EXAMPLE 13 Horse Race Payoff

At a horse race, the odds on Blue Moon are given as 7 to 2. If you bet $10 and Blue Moon wins, how much will you gain?

EXAMPLE 1 Flaw in the Chevalier's Thinking

Consider the Chevalier's first bet, in which he guessed that the probability of rolling at least one 6 in four rolls would be four times the single-roll probability of 1/6, or 4/6. If we extended the same logic, what would we find for the probability of rolling at least one 6 in five rolls and in six rolls? Explain how this extension proves that his logic was incorrect.

And Probability: Independent Events

Two events are **independent** if the occurrence of one event does not affect the probability of the other event. Consider two independent events A and B with probabilities $P(A)$ and $P(B)$. The ***and*** **probability** that A and B both occur is

$$P(A \text{ and } B) = P(A) \times P(B)$$

This principle can be extended to any number of independent events. For example, the probability of A, B, and a third independent event C occurring together is

$$P(A \text{ and } B \text{ and } C) = P(A) \times P(B) \times P(C)$$

EXAMPLE 2 Three Coins

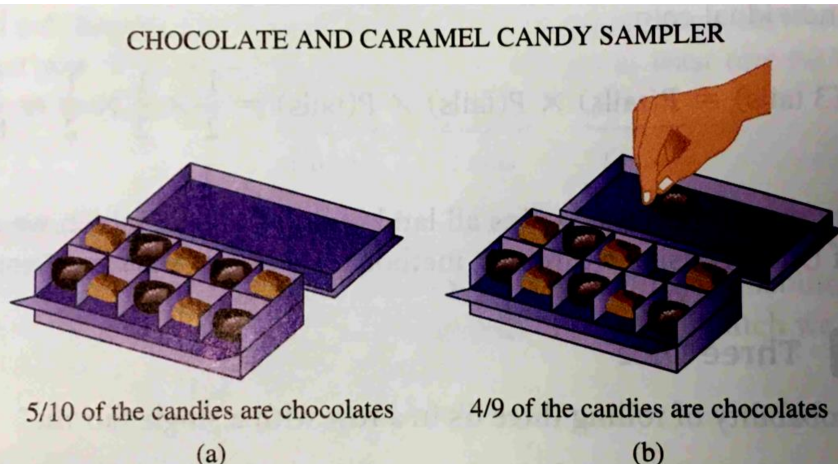
Suppose you toss three fair coins. What is the probability of getting three tails?

EXAMPLE 3 Three Dice

What is the probability of rolling three 6's in a row with a single fair die?

EXAMPLE 4 Consecutive Floods

Find the probability that a 100-year flood (a flood with a 0.01 probability of occurring in a given year) will hit the same city in two consecutive years. Assume that a flood in one year does not affect the likelihood of a flood in the next year.



IGURE 7.9 A box of candy contains five chocolates and five caramels. (a) If you choose one candy at andom, the probability of getting a chocolate is 5/10, or 1/2. (b) If the first candy is a chocolate and you at it, then there are only four chocolates left among the nine remaining candies. Therefore, the probability of getting another chocolate on the second draw is 4/9.

And Probability: Dependent Events

Two events are **dependent** if the occurrence of one event affects the probability of the other event. The ***and* probability** that dependent events A and B both occur is

$$P(A \text{ and } B) = P(A) \times P(B \text{ given } A)$$

where "$P(B \text{ given } A)$" means the probability of event B given the occurrence of event A. This principle can be extended to any number of dependent events. For example, the *and* probability of three dependent events A, B, and C is

$$P(A \text{ and } B \text{ and } C) = P(A) \times P(B \text{ given } A) \times P(C \text{ given } A \text{ and } B)$$

EXAMPLE 5 Bingo

The game of Bingo involves drawing labeled buttons from a bin at random, without replacing those drawn. There are 75 buttons, 15 for each of the letters B, I, N, G, and O. What is the probability of drawing two B buttons in the first two selections?

EXAMPLE 6 Jury Selection

A three-person jury must be selected at random from a pool that has 6 men and 6 women. What is the probability of selecting an all-male jury?

Either/Or Probability: Non-overlapping Events

Two events are **non-overlapping** if they cannot occur together. If A and B are non-overlapping events, the probability that either A or B occurs is

$$P(A \text{ or } B) = P(A) + P(B)$$

This principle can be extended to any number of non-overlapping events. For example, the probability that either event A, event B, or event C occurs is

$$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)$$

EXAMPLE Either/Or Dice

Suppose you roll a single die. What is the probability of rolling either a 2 or a 3?

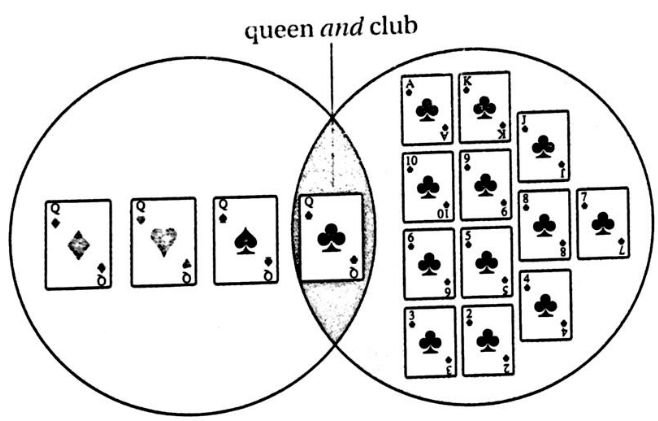


FIGURE 7.11 Venn diagram for overlapping events. One circle represents the queens in a deck of cards and the other represents the clubs. The overlap region contains the queen of clubs, which belongs in both circles.

in both circles.

Either/Or Probability: Overlapping Events

Two events are **overlapping** if they *can* occur together. If A and B are overlapping events, the probability that either A or B occurs is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

EXAMPLE 8 **Democrats and Women**

There are eight people in a room: two Democratic men, two Republican men, two Democratic women, and two Republican women. If you select one person at random from this room, what is the probability that you will select *either* a woman or a Democrat?

The *At Least Once* Rule (for Independent Events)

Suppose the probability of an event A occurring in one trial is $P(A)$. If all trials are independent, the probability that event A occurs *at least once* in n trials is

$$\begin{aligned} P(\text{at least one event } A \text{ in } n \text{ trials}) &= 1 - P(\text{no } A \text{ in } n \text{ trials}) \\ &= 1 - [P(\text{not } A)]^n \end{aligned}$$

EXAMPLE **At Least One Head with Three Coins**

Use the *at least once* rule to find the probability of getting at least one head when you toss three coins.

EXAMPLE 10 **100-Year Flood**

Find the probability that a region will experience *at least one* 100-year flood (a flood that has a 0.01 chance of occurring in any given year) during the next 100 years. Assume flood events are independent from year to year.

EXAMPLE 11 **Lottery Chances**

You purchase 10 lottery tickets, for which the probability of winning some type of prize on a single ticket is 1 in 10. What is the probability that you will have at least one winning ticket among the 10 tickets?

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