### Definitions

**Logic** is the study of the methods and principles of reasoning.

An **argument** uses a set of facts or assumptions, called **premises**, to support a **conclusion.**

A **fallacy** is a deceptive argument—an argument in which the conclusion is not well supported by the premises.

## EXAMPLE 1 Appeal to Popularity

"Ford makes the best pickup trucks in the world. More people drive Ford pickups than any other light truck."

## EXAMPLE 2 False Cause

"I placed the quartz crystal on my forehead, and in five minutes my headache was gone. The crystal made my headache go away."

## EXAMPLE 3 Appeal to Ignorance

"Scientists have not found any concrete evidence of aliens visiting Earth. Therefore, anyone who claims to have seen a UFO must be hallucinating."

**EXAMPLE 4** Hasty Generalization

"Two cases of childhood leukemia have occurred along the street where the high-voltage power lines run. The power lines must be the cause of these illnesses."

**EXAMPLE 5** Limited Choice

"You don't support the President, so you are not a patriotic American."

**EXAMPLE 6** Appeal to Emotion

In ads for Michelin tires, a picture of a baby is shown with the words "because so much is riding on your tires."

**EXAMPLE 7** Personal Attack

Gwen: *You should stop drinking because it's hurting your grades, endangering people when you drink and drive, and destroying your relationship with your family.*

Merle: *I've seen you drink a few too many on occasion yourself!*

**EXAMPLE 8** Circular Reasoning

"Society has an obligation to provide health insurance because health care is a right of citizenship."

## EXAMPLE 9 Diversion (Red Herring)

"We should not continue to fund cloning research because there are so many ethical issues involved. Strong ethical foundations are necessary to the proper functioning of society, so we cannot afford to have too many ethical loose ends."

## EXAMPLE 10 Straw Man

Suppose that the mayor of a large city proposes decriminalizing drug possession in order to reduce overcrowding in jails and save money on enforcement. His challenger in the upcoming election says, "The mayor doesn't think there's anything wrong with drug use, but I do."

### Five Steps to Evaluating Media Information

1. **Consider the source.** Are you seeing the information from its original source, and if not, can you track down the original source? Does this source have credibility on this issue? Be especially careful to make sure this source is what you think it is; many websites that spread fake news make themselves look like the legitimate sites of real news organizations.
2. **Check the date.** Can you determine when the information was written? Is it still relevant, or is it outdated?
3. **Validate accuracy.** Can you validate the information from other sources (such as major news websites)? Do you have good reason to believe it is accurate? Does it contain anything that makes you suspicious?
4. **Watch for hidden agendas.** Is the information presented fairly and objectively, or is it manipulated to serve some particular or hidden agenda?
5. **Don't miss the big picture.** Even if a piece of media information passes all the above tests, step back and consider whether it makes sense. For example, does it conflict with other things you think are true, and if so, how can you resolve the conflict?

1B

**Definition**

A **proposition** makes a claim (either an assertion or a denial) that may be either true or false. It must have the structure of a complete sentence.

**Definitions**

Any proposition has two possible **truth values:** T = true or F = false.

The **negation** of a proposition $p$ is another proposition that makes the opposite claim of $p$. It is written *not* $p$ (or $\sim p$), and its truth value is opposite to that of $p$.

A **truth table** is a table with a row for each possible set of truth values for the propositions being considered.

## EXAMPLE 1 Negation

Find the negation of the proposition *Amanda is the fastest runner on the team*. If the negation is false, is Amanda really the fastest runner on the team?

## EXAMPLE 2 Radiation and Health

After reviewing data showing an association between low-level radiation and cancer among older workers at the Oak Ridge National Laboratory, a health scientist from the University of North Carolina (Chapel Hill) was asked about the possibility of a similar association among younger workers at another national laboratory. He was quoted as saying (*Boulder Daily Camera*):

> *My opinion is that it's unlikely that there is no association.*

Does the scientist think there is an association between low-level radiation and cancer among younger workers?

## EXAMPLE 3 The Miranda Ruling

If you've ever watched a crime show, you are familiar with the Miranda rights that law enforcement officers recite to suspects ("You have the right to remain silent, ..."). These rights stem from a 1966 decision of the U.S. Supreme Court (*Miranda v. State of Arizona*), which the Court revisited in a case in 2000 (*Dickerson v. United States*). In his majority opinion, Chief Justice William Rehnquist wrote:

> *... [legal] principles weigh heavily against overruling [Miranda].*

Based on this statement, did the Court support or oppose the original Miranda decision?

### The Logic of *And*

Given two propositions *p* and *q*, the statement *p and q* is called their **conjunction.** It is true only if *p* and *q* are *both* true.

## EXAMPLE 4 *And* Statements

Evaluate the truth value of the following two statements.

**a.** The capital of France is Paris and Antarctica is cold.
**b.** The capital of France is Paris and the capital of America is Madrid.

## EXAMPLE 5 Triple Conjunction

Suppose you are given three individual propositions *p*, *q* and *r*. Make a truth table for the conjunction *p and q and r*. Under what circumstances is the conjunction true?

### Two Types of *Or*

The word *or* can be interpreted in two distinct ways:

- An **inclusive *or*** means "either or both."
- An **exclusive *or*** means "one or the other, but not both."

In everyday life, we determine whether an *or* statement is inclusive or exclusive by its context. But in logic, we assume that *or* is inclusive unless told otherwise.

## EXAMPLE 6 Inclusive or Exclusive?

Kevin's insurance policy states that his house is insured for earthquake, fire, or robbery. Imagine that a major earthquake levels much of his house, the rest burns in a fire, and his remaining valuables are looted in the aftermath. Would Kevin prefer that the *or* in his insurance policy be inclusive or exclusive? Why?

### The Logic of *Or*

Given two propositions, $p$ and $q$, the statement $p$ *or* $q$ is called their **disjunction.** In logic, we assume that *or* is *inclusive*, so the disjunction is true if either or both propositions are true, and false only if *both* propositions are false.

## EXAMPLE 7 Smart Cows?

Consider the statement *Airplanes can fly or cows can read.* Is it true?

### The Logic of *if... then*

A statement of the form *if p, then q* is called a **conditional proposition** (or *implication*). Proposition $p$ is called the **hypothesis** and proposition $q$ is called the **conclusion**. The conditional *if p, then q* is true in all cases except the case in which $p$ is true and $q$ is false.

## EXAMPLE 8 Conditional Truths

Evaluate the truth of the statement *if* $2 + 2 = 5$, *then* $3 + 3 = 4$.

### Alternative Phrasings of Conditionals

The following are common alternative ways of stating *if p, then q*:

| | | |
|---|---|---|
| *p is sufficient for q* | *p will lead to q* | *p implies q* |
| *q is necessary for p* | *q if p* | *q whenever p* |

## EXAMPLE 9 Rephrasing Conditional Propositions

Recast each of the following statements in the form *if p, then q*.

**a.** A rise in sea level will devastate Florida.
**b.** A red tag on an item is sufficient to indicate it's on sale.
**c.** Eating vegetables is necessary for good health.

**Variations on the Conditional**

| Name | Form | Example |
|---|---|---|
| Conditional | *if p, then q* | If you are sleeping, then you are breathing. |
| Converse | *if q, then p* | If you are breathing, then you are sleeping. |
| Inverse | *if not p, then not q* | If you are not sleeping, then you are not breathing. |
| Contrapositive | *if not q, then not p* | If you are not breathing, then you are not sleeping. |

## EXAMPLE 10 Logical Equivalence

Consider the true statement *If a creature is a whale, then it is a mammal.* Write its converse, inverse, and contrapositive. Evaluate the truth of each statement. Which statements are logically equivalent?

### Definition

Two statements are **logically equivalent** if they share the same truth values: If one is true, so is the other, and if one is false, so is the other.

1C

**Definitions**

A **set** is a collection of objects; the individual objects are the **members** of the set. We write sets by listing their members within a pair of braces, { }. If there are too many members to list, we use three dots, . . . , to indicate a continuing pattern.

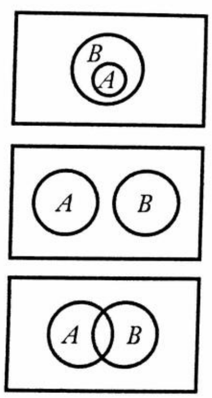
**EXAMPLE 1** **Set Notation**

Use braces to write the contents of each of the following sets:

**a.** the set of countries larger in land area than the United States
**b.** the set of years of the Cold War, generally taken to have started in 1945 and ended in 1991
**c.** the set of natural numbers greater than 5

## Set Relationships and Venn Diagrams

Two sets *A* and *B* may be related in three basic ways:




- *A* may be a **subset** of *B* (or vice versa), meaning that all members of *A* are also members of *B*. The Venn diagram for this case shows the circle for *A* inside the circle for *B*.
- *A* may be **disjoint** from *B*, meaning that the two sets have no members in common. The Venn diagram for this case consists of separated circles that do not touch.
- *A* and *B* may be **overlapping** sets, meaning that the two sets share some of the same members. The Venn diagram for this case consists of two overlapping circles. We also use overlapping circles for cases in which the two sets might share common members.

## EXAMPLE 2 Venn Diagrams

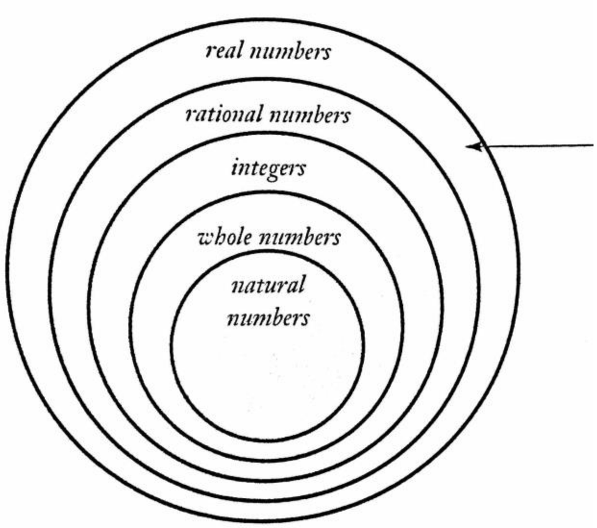
Describe the relationship between the given pairs of sets, and draw a Venn diagram showing this relationship. Interpret all the regions of the Venn diagram.

a. *Democrats* and *Republicans* (party affiliations)
b. *Nobel Prize winners* and *Pulitzer Prize winners*



## EXAMPLE 3 Sets of Numbers

Draw a Venn diagram showing the relationships among the sets of natural numbers, whole numbers, integers, rational numbers, and real numbers. Where are irrational numbers found in this diagram? (If you've forgotten the meanings of these number sets, see the following Brief Review.)



## The Four Standard Categorical Propositions

| Form | Example | Subject Set (*S*) | Predicate Set (*P*) |
|---|---|---|---|
| All *S* are *P*. | All whales are mammals. | *whales* | *mammals* |
| No *S* are *P*. | No fish are mammals. | *fish* | *mammals* |
| Some *S* are *P*. | Some doctors are women. | *doctors* | *women* |
| Some *S* are not *P*. | Some teachers are not men. | *teachers* | *men* |

## EXAMPLE 4 Interpreting Venn Diagrams

Answer the following questions based only on the information provided in the Venn diagrams. That is, don't consider any prior knowledge you have about the sets.

**a.** Based on Figure 1.16, can you conclude that some mammals are not whales?
**b.** Based on Figure 1.17, is it possible that some mammals are fish?
**c.** Based on Figure 1.18, is it possible that all doctors are women?
**d.** Based on Figure 1.19, is it possible that no men are teachers?

## EXAMPLE 5 Rephrasing in Standard Form

Rephrase each of the following statements in one of the four standard forms for categorical propositions. Then draw the Venn diagram.

**a.** Some birds can fly.
**b.** Elephants never forget.

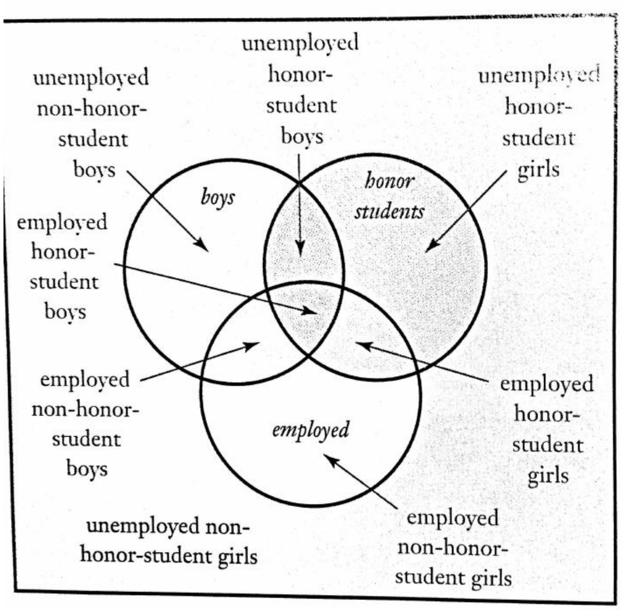



**FIGURE 1.21** A Venn diagram for three overlapping sets has $2^3 = 8$ regions.

## EXAMPLE 6 Recording Data in a Venn Diagram

You hire an assistant to help you with the study of teenage employment described above. He focuses on a small group of teenagers who are all enrolled in the same school. He reports the following facts about this group:

- Some of the honor-student boys are unemployed.
- Some of the non-honor-student girls are employed.

Put Xs in the appropriate places in Figure 1.21 to indicate the regions that you can be sure have members. Based on this report, do you know whether any of the school's honor-student girls are unemployed? Why or why not?

## EXAMPLE 7 Color Monitors

Color television and computer monitors make all the colors you see by combining *pixels* (short for "picture elements") that display just three colors: red, green, and blue. In pairs, these combinations of colors give the following results (with the two colors at equal strength):

| Combination | Result |
|---|---|
| Red-green | Yellow |
| Red-blue | Purple (or magenta) |
| Blue-green | Light blue (or cyan) |

## EXAMPLE 8 Smoking and Pregnancy

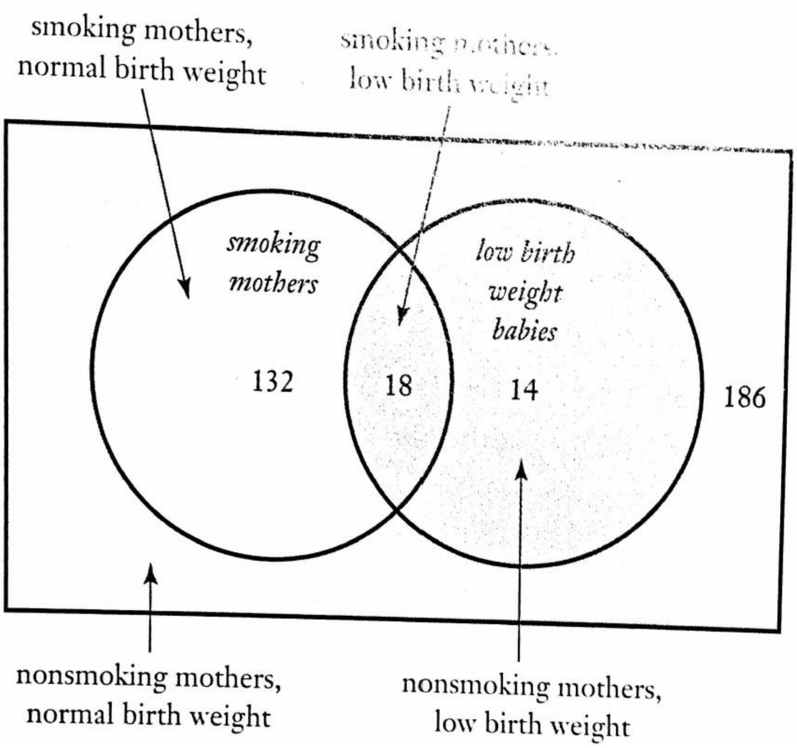
Consider the study summarized in Table 1.1, which was designed to learn whether a pregnant mother's status as a smoker or nonsmoker affects whether she delivers a baby with a low or a normal birth weight. This table is an example of what we call a **two-way table**, because it shows two variables: *mother's smoking status* and *baby's birth weight status*. The table caption tells us that the study involved 350 births, and the four data cells tell us the number of births with each of the four possible combinations of the two variables.

**TABLE 1.1 Distribution of 350 Births by Birth Weight Status and Mother's Smoking Status**

| | | Baby's Birth Weight Status | |
|---|---|---|---|
| | | **Low Birth Weight** | **Normal Birth Weight** |
| **Mother's Smoking Status** | **Smoker** | 18 | 132 |
| | **Nonsmoker** | 14 | 186 |

*Source:* Data from U.S. National Center for Health Statistics.

**a.** Make a list summarizing the four key facts shown in the table.
**b.** Draw a Venn diagram to represent the table data.
**c.** Based on the Venn diagram, briefly summarize the results of the study.



**EXAMPLE 9** **Three Sets with Numbers—Blood Types**

Human blood is often classified according to whether three antigens, A, B, and Rh, are present or absent. Blood type is stated first in terms of the antigens A and B: Blood containing only A is called type A, blood containing only B is called type B, blood containing both A and B is called type AB, and blood containing neither A nor B is called type O. The presence or absence of Rh is indicated by adding the word "positive" (present) or "negative" (absent) or the equivalent symbol (+ or −). Table 1.2 shows the eight blood types that result and the percentage of people with each type in the U.S. population. Draw a Venn diagram to illustrate these data.

**TABLE 1.2** **Blood Types in U.S. Population**

| Blood Type | Percentage of Population |
|---|---|
| A positive | 34% |
| B positive | 8% |
| AB positive | 3% |
| O positive | 35% |
| A negative | 8% |
| B negative | 2% |
| AB negative | 1% |
| O negative | 9% |

1D

**Definition**

An **inductive argument** makes a case for a general conclusion from more specific premises.

A **deductive argument** makes a case for a specific conclusion from more general premises.

## Evaluating an Inductive Argument

An inductive argument cannot prove its conclusion true, so we evaluate it only in terms of its **strength.** An argument is strong if it makes a compelling case for its conclusion. It is weak if its conclusion is not well supported by its premises.

### EXAMPLE 1 Hit Movie

A movie director tells her producer (who pays for the movie to be made) not to worry—her film will be a hit. As evidence, she cites the following facts: She's hired big stars for the lead roles, she has a great advertising campaign planned, and it's a sequel to her last hit movie. Is this an inductive or deductive argument? Evaluate its strength.

### EXAMPLE 2 Earthquake

Evaluate the following argument, and discuss the truth of its conclusion.

*Geological evidence shows that, for thousands of years, the San Andreas Fault has suffered a major earthquake at least once every hundred years. Therefore, we should expect another earthquake on the fault during the next one hundred years.*

### Evaluating a Deductive Argument

We apply two criteria when evaluating a deductive argument:

- The argument is **valid** if its conclusion follows necessarily from its premises, regardless of the truth of those premises or conclusions.
- The argument is **sound** if it is valid *and* its premises are all true.

### A Venn Diagram Test of Validity

To test the validity of a deductive argument with a Venn diagram:

1. Draw a Venn diagram that represents all the information contained in the premises.
2. Check to see whether the Venn diagram confirms the conclusion. If it does, then the argument is valid. Otherwise, the argument is not valid.

**EXAMPLE 3** Invalid Argument

Evaluate the validity and soundness of the following argument.

| | |
|---|---|
| Premise: | All fish live in water. |
| Premise: | Whales are not fish. |
| Conclusion: | Whales do not live in water. |

**EXAMPLE 4** Invalid but True Conclusion

Evaluate the validity and soundness of the following argument.

| | |
|---|---|
| Premise: | All 20th-century U.S. Presidents were men. |
| Premise: | John Kennedy was a man. |
| Conclusion: | John Kennedy was a 20th-century U.S. President. |

**Four Basic Conditional Arguments**

| | Affirming the Hypothesis* | Affirming the Conclusion | Denying the Hypothesis | Denying the Conclusion** |
|---|---|---|---|---|
| **Structure** | If $p$, then $q$.<br>$p$ is true.<br>———<br>$q$ is true. | If $p$, then $q$.<br>$q$ is true.<br>———<br>$p$ is true. | If $p$, then $q$<br>$p$ is not true.<br>———<br>$q$ is not true. | If $p$, then $q$.<br>$q$ is not true.<br>———<br>$p$ is not true. |
| **Validity** | Valid | Invalid | Invalid | Valid |

*Also known by the Latin term *modus ponens*.
**Also known by the Latin term *modus tollens*.

## EXAMPLE 5 Affirming the Hypothesis (Valid)

Use a Venn diagram test to show that the argument concerning Carlos and Chicago (which affirms the hypothesis) is valid.

## EXAMPLE 6 Affirming the Conclusion (Invalid)

Use a Venn diagram to test the validity of the following argument, which affirms the conclusion.

Premise: If an employee is regularly late, then the employee will be fired.
Premise: Sharon was fired.
Conclusion: Sharon was regularly late.

lls us that

**EXAMPLE 7** Denying the Hypothesis (Invalid)

Use a Venn diagram to test the validity of the following argument, which has the form called denying the hypothesis.

Premise: If you liked the book, then you loved the movie.
Premise: You did not like the book.
Conclusion: You did not love the movie.

**EXAMPLE 8** Denying the Conclusion (Valid)

Use a Venn diagram to test the validity of the following argument, which has the form denying the conclusion.

Premise: A narcotic is habit-forming.
Premise: Aspirin is not habit-forming.
Conclusion: Aspirin is not a narcotic.

## Deductive Arguments with a Chain of Conditionals

Another common type of deductive argument involves a chain of three or more conditionals. Such arguments have the following form:

Premise: If $p$, then $q$.
Premise: If $q$, then $r$.
Conclusion: If $p$, then $r$.

This particular chain of conditionals is valid: If $p$ implies $q$ and $q$ implies $r$, it must be true that $p$ implies $r$.

**EXAMPLE 9** A Chain of Conditionals

Determine the validity of this argument: "If elected to the school board, Maria Lopez will force the school district to raise academic standards, which will benefit my children's education. Therefore, my children will benefit if Maria Lopez is elected."

**EXAMPLE 10** Invalid Chain of Conditionals

Determine the validity of the following argument: "We agreed that if you shop, I make dinner. We also agreed that if you take out the trash, I make dinner. Therefore, if you shop, you should take out the trash."

**EXAMPLE 11** Inductively Testing a Mathematical Rule

Test the following rule: For all numbers $a$ and $b$, $a \times b = b \times a$.

**EXAMPLE 12** Invalidating a Proposed Rule

Suppose you cannot recall whether adding the same amount to both the numerator and the denominator (top and bottom) of a fraction such as $\frac{2}{3}$ is legitimate. That is, you are wondering whether it is true that, for all numbers $a$,

$$\frac{2}{3} \stackrel{?}{=} \frac{2+a}{3+a}$$