Brief Review Percentages

The words *per cent* mean "per 100" or "divided by 100." We use the symbol % as shorthand for *per cent*. For example, we read 50% as "50 percent" and its meaning is

$$50\% = \frac{50}{100} = 0.5$$

More generally, for a number P,

$$P\% = \frac{P}{100}$$

For example:

$$100\% = \frac{100}{100} = 1 \qquad 200\% = \frac{200}{100} = 2$$

$$350\% = \frac{350}{100} = 3.5$$

Note that multiplying a number by 100% does not change its value, because 100% is just another way of writing 1. For example, multiplying 1.25 by 100% gives

$$1.25 \times 100\% = 125\%$$

That is, 125% is just another way of writing 1.25. These basic ideas lead to the following rules for converting between percentages and decimals or common fractions.

- **To convert a percentage to a common fraction:** Replace the % symbol with division by 100; simplify the fraction if necessary.

  Example: $25\% = \frac{25}{100} = \frac{1}{4}$

- **To convert a percentage to a decimal:** Drop the % symbol and divide by 100 (equivalent to moving the decimal point two places to the left).

  Example: $25\% = \frac{25}{100} = 0.25$

- **To convert a decimal to a percentage:** Multiply by 100 (equivalent to moving the decimal point two places to the right) and insert the % symbol.

  Example: $0.43 = \frac{43}{100} = 43\%$

- **To convert a common fraction to a percentage:** First convert the common fraction to decimal form, using a calculator if necessary; then convert the decimal to a percentage.

  Example: $\frac{1}{5} = 0.2 = 20\%$

▶ **Now try Exercises 15–30.**

EXAMPLE 1 Presidential Survey

An opinion poll finds that 35% of 1069 people surveyed said that the President is doing a good job. How many said the President is doing a good job?

Absolute and Relative Change

The **absolute change** describes the actual increase or decrease from a reference value to a new value:

$$\text{absolute change} = \text{new value} - \text{reference value}$$

The **relative change** is the size of the absolute change in comparison to the reference value and can be expressed as a percentage:

$$\text{relative change} = \frac{\text{new value} - \text{reference value}}{\text{reference value}} \times 100\%$$

EXAMPLE 2 **Stock Price Rise**

During a 6-month period, Lunar Industry's stock doubled in price from $7 to $14. What were the absolute and relative changes in the stock price?

EXAMPLE 3 **World Population Growth**

Estimated world population increased from 2.7 billion in 1953 to 7.5 billion in 2018. Describe the absolute and relative change in world population over this 65-year period.

EXAMPLE 4 **Depreciating a Computer**

You bought a new laptop computer three years ago for $1000. Today, it is worth only $300. Describe the absolute and relative change in the laptop's value.

Absolute and Relative Difference

The **absolute difference** is the actual difference between the compared value and the reference value:

$$\text{absolute difference} = \text{compared value} - \text{reference value}$$

The **relative difference** describes the size of the absolute difference in comparison to the reference value and can be expressed as a percentage:

$$\text{relative difference} = \frac{\text{compared value} - \text{reference value}}{\text{reference value}} \times 100\%$$

EXAMPLE 5 **Income Comparison**

Recent data showed that California ranked first among the 50 states in average income, at about $68,900 per person, and West Virginia ranked last at $46,600 per person.

a. How much lower is average income in West Virginia than in California?
b. How much higher is average income in California than in West Virginia?

Of versus *More Than* (or *Less Than*)

- If the new or compared value is *P*% *more than* the reference value, it is $(100 + P)\%$ *of* the reference value.
- If the new or compared value is *P*% *less than* the reference value, it is $(100 - P)\%$ *of* the reference value.

EXAMPLE 6 **Income Difference**

Carol earns 200% more than William. What is Carol's income as a percentage *of* William's? How many times as large as William's income is Carol's?

Solution We use the rule that *P% more than* means $(100 + P)\%$ *of*. Because Carol's income is 200% more than William's, we set $P = 200$. Therefore, Carol's income is $(100 + 200)\% = 300\%$ *of* William's income. Because $300\% = 3$, Carol earns 3 times as much as William.

EXAMPLE 7 **Sale!**

A store is having a "25% off" sale. How does an item's sale price compare to its original price?

Percentage Points versus Percent (or %)

When you see a change or difference expressed in *percentage points,* you can assume it is an *absolute* change or difference. If it is expressed with the % sign or the word *percent*, it should be a *relative* change or difference.

EXAMPLE 8 **Newspaper Readership Declines**

According to Pew Research Center polls, the percentage of adults who regularly read a daily newspaper fell from about 54% in 2004 to about 38% in 2017. Describe this change in newspaper readership.

Solving Percentage Problems

Given that some final (or compared) value is P% *more than* an initial (or reference) value, calculations will be easier if you first convert the *more than* statement to an *of* statement, which you can write as multiplication:

$$\text{final value} = (100 + P)\% \times \text{initial value}$$

Use the equation in the above form if you are given P and the initial value. If you are given the final value and want to find the initial value, then rearrange the above equation:

$$\text{initial value} = \frac{\text{final value}}{(100 + P)\%}$$

If the final value is *less than* the initial value, then use $(100 - P)$ instead of $(100 + P)$ in the above equations.

EXAMPLE 10 **Tax Calculations**

a. You purchase a t-shirt with a labeled (pre-tax) price of $17. The local sales tax rate is 5%. What is your final cost (including tax)?

b. Your receipt shows that you paid $19.26 for a phone case, tax included. The local sales tax rate is 7%. What was the labeled (pre-tax) price of the case?

c. You are eligible for a 15% student discount for basketball tickets that otherwise cost $55. How much will you pay?

EXAMPLE 13 **Tax Cuts**

A politician promises, "If elected, I will cut your taxes by 20% for each of the first three years of my term, for a total cut of 60%." Evaluate the promise.

EXAMPLE 14 **Impossible Sale**

A store advertises that it will take "150% off" the price of all merchandise. What should happen when you go to the checkout to buy a $500 item?

3B

Definition

Scientific notation is a format in which a number is expressed as a number *between* 1 and 10 multiplied by a power of 10.

EXAMPLE 1 Numbers in Scientific Notation

Rewrite each of the following statements using scientific notation.

a. Total spending in the new federal budget is $4,200,000,000,000.
b. The diameter of a hydrogen nucleus is about 0.000000000000001 meter.

Working with Scientific Notation

Converting to Scientific Notation

To convert a number from ordinary notation to scientific notation:

Step 1. Move the decimal point to come after the *first* nonzero digit.

Step 2. For the power of 10, use the number of places the decimal point moves; the power is *positive* if the decimal point moves to the left and *negative* if it moves to the right.

Examples:

$$3042 \xrightarrow[\text{3 places to left}]{\text{decimal moves}} 3.042 \times 10^3$$

$$0.00012 \xrightarrow[\text{4 places to right}]{\text{decimal moves}} 1.2 \times 10^{-4}$$

$$226 \times 10^2 \xrightarrow[\text{2 places to left}]{\text{decimal moves}} (2.26 \times 10^2) \times 10^2 = 2.26 \times 10^4$$

Converting from Scientific Notation

To convert a number from scientific notation to ordinary notation:

Step 1. The power of 10 indicates how many places to move the decimal point; move it to the *right* if the power of 10 is positive and to the *left* if it is negative.

Step 2. If moving the decimal point creates any open places, fill them with zeros.

Examples:

$$4.01 \times 10^2 \xrightarrow[\text{2 places to right}]{\text{move decimal}} 401$$

$$3.6 \times 10^6 \xrightarrow[\text{6 places to right}]{\text{move decimal}} 3{,}600{,}000$$

$$5.7 \times 10^{-3} \xrightarrow[\text{3 places to left}]{\text{move decimal}} 0.0057$$

Multiplying or Dividing with Scientific Notation

Multiplying or dividing numbers expressed in scientific notation simply requires operating on the powers of 10 and the other parts of the numbers separately.

Examples:

$$(6 \times 10^2) \times (4 \times 10^5) = (6 \times 4) \times (10^2 \times 10^5) = 24 \times 10^7 = 2.4 \times 10^8$$

$$\frac{4.2 \times 10^{-2}}{8.4 \times 10^{-5}} = \frac{4.2}{8.4} \times \frac{10^{-2}}{10^{-5}} = 0.5 \times 10^{-2-(-5)} = 0.5 \times 10^3 = 5 \times 10^2$$

Note that, in both examples, we first found an answer in which the number multiplied by a power of 10 was *not* between 1 and 10. We then followed the process for converting the final answer into scientific notation.

Addition and Subtraction with Scientific Notation

In general, we must write numbers in ordinary notation before adding or subtracting.

Examples:

$$(3 \times 10^6) + (5 \times 10^2) = 3{,}000{,}000 + 500 = 3{,}000{,}500 = 3.0005 \times 10^6$$

$$(4.6 \times 10^9) - (5 \times 10^8) = 4{,}600{,}000{,}000 - 500{,}000{,}000 = 4{,}100{,}000{,}000 = 4.1 \times 10^9$$

When both numbers have the *same* power of 10, we can factor out the power of 10 first.

Examples:

$$(7 \times 10^{10}) + (4 \times 10^{10}) = (7 + 4) \times 10^{10} = 11 \times 10^{10} = 1.1 \times 10^{11}$$

$$(2.3 \times 10^{-22}) - (1.6 \times 10^{-22}) = (2.3 - 1.6) \times 10^{-22} = 0.7 \times 10^{-22} = 7.0 \times 10^{-23}$$

▸ **Now try Exercises 15–22.**

Definition

An **order of magnitude** estimate specifies only a broad range of values, usually within one or two powers of ten, such as "in the ten thousands" or "in the millions."

EXAMPLE 3 Order of Magnitude of Ice Cream Spending

Make an order of magnitude estimate of total annual spending on ice cream in the United States.

EXAMPLE 4 U.S. vs. World Energy Consumption

Compare the U.S. population to the world population and U.S. energy consumption to world energy consumption. What does this tell you about energy usage by Americans?

TABLE 3.1 Selected Energy Comparisons

Item	Energy (joules)
Energy released by metabolism of 1 average candy bar	1×10^{6}
Energy needed for 1 hour of running (adult)	4×10^{6}
Energy released by burning 1 liter of oil	1.2×10^{7}
Electrical energy used in an average home daily	5×10^{7}
Energy released by burning 1 kilogram of coal	1.6×10^{9}
Energy released by fission of 1 kilogram of uranium-235	5.6×10^{13}
Energy released by fusion of hydrogen in 1 liter of water	6.9×10^{13}
U.S. annual energy consumption	1.0×10^{20}
World annual energy consumption	5.7×10^{20}
Annual energy generation of Sun	1×10^{34}

meter and 1000 meters in 1 kilometer), so a scale where 1 centimeter represents 1 kilometer can be described as a **scale ratio** of 1 to 100,000 (or 1/100,000).

FIGURE 3.1 The miniruler at the lower left acts as the map scale. In this case, the length of the upper segment represents 2000 feet in the city, while the slightly shorter lower segment represents 500 meters.

EXAMPLE 6 Scale Ratio

A city map states, "One inch represents one mile." What is the scale ratio for this map?

EXAMPLE 7 Earth and Sun

The distance from the Earth to the Sun is about 150 million kilometers. The diameter of the Sun is about 1.4 million kilometers, and the equatorial diameter of the Earth is about 12,760 kilometers. Put these numbers in perspective by using a scale model of the solar system with a 1 to 10 billion scale.

EXAMPLE 8 Distances to the Stars

The distance from the Earth to the nearest stars besides the Sun (the three stars of the Alpha Centauri system) is about 4.3 light-years. On the 1 to 10 billion scale of Example 7, how far are these stars from the Earth? (*Note:* Recall that 1 light-year $\approx$ 9.46 $\times$ 10^{12} km.)

EXAMPLE 9 Timeline

Human civilization, at least since the time of ancient Egypt, is on the order of 5000 years old. The age of the Earth is on the order of 5 billion years. Suppose we use the length of a football field, or about 100 meters, as a timeline to represent the age of the Earth. If we put the birth of the Earth at the start of the timeline, how far from the line's end would human civilization begin?