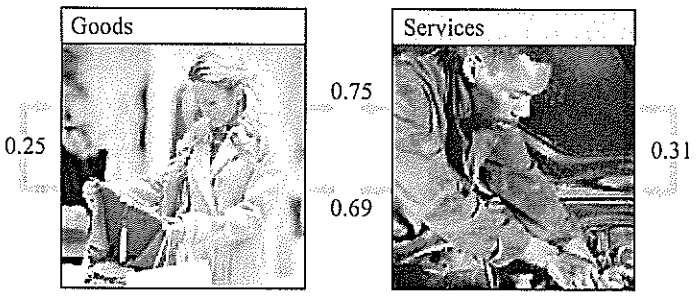
Student: _____ **Instructor:** Jeremiah Dyke
Date: _____ **Course:** MTH 285 Linear Algebra (1) **Assignment:** Midterm

1. Suppose an economy has only two sectors: Goods and Services. Each year, Goods sells 75% of its outputs to Services and keeps the rest, while Services sells 69% of its output to Goods and retains the rest. Find equilibrium prices for the annual outputs of the Goods and Services sectors that make each sector's income match its expenditures.



Denote the prices (that is, dollar values) of the total annual outputs of the Goods and Services sectors by p_G and p_S, respectively.

If $p_S = \$1000$, then $p_G = \$$_____. (Type an integer or a decimal.)

If $p_S = \$75$, then $p_G = \$$_____. (Type an integer or a decimal.)

ID: 1.6.1

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2. Suppose an economy consists of the Coal, Electric, and Steel sectors. Denote the prices (that is, dollar values) of the total annual outputs of the Coal, Electric, and Steel sectors by p_C, p_E, and p_S, respectively. Suppose the general solution to find equilibrium prices that make each sector's income match its expenditures is $p_C = 0.89p_S$, $p_E = 0.84p_S$, and p_S is free. One set of equilibrium prices for this economy is $p_C = \$89$, $p_E = \$84$, and $p_S = \$100$. Find another set. Suppose the same economy used Japanese yen instead of dollars to measure the values of the various sector's output. Would this change the problem in any way? Discuss.

If $p_S = \$200$, then $p_C = \$$_____ and $p_E = \$$_____. (Type integers or decimals.)

How would changing the unit of measurement to Japanese yen change this problem?

- ○ **A.** It has the same effect as multiplying all equilibrium prices by a constant. The ratios of the prices will change, but the prices will remain the same.
- ○ **B.** It has the same effect as multiplying all equilibrium prices by their reciprocal and some constant. The prices will change, but the ratios of the prices will remain the same.
- ○ **C.** It has the same effect as multiplying all equilibrium prices by a constant. The prices will change, but the ratios of the prices will remain the same.
- ○ **D.** It has the same effect as multiplying all equilibrium prices by their reciprocal and some constant. The prices and the ratios of the prices will change.

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3. Consider an economy with three sectors, Chemicals & Metals, Fuels & Power, and Machinery. Chemicals sells 20% of its output to Fuels and 60% to Machinery and retains the rest. Fuels sells 80% of its output to Chemicals and 10% to Machinery and retains the rest. Machinery sells 20% of its output to Chemicals and 40% to Fuels and retains the rest. Complete parts (a) through (c) below.

a. Construct the exchange table for this economy.

Distribution of Output from:			
Chemicals	Fuels	Machinery	Purchased by:
_____	_____	_____	Chemicals
_____	_____	_____	Fuels
_____	_____	_____	Machinery

(Type integers or decimals.)

b. Develop a system of equations that leads to prices at which each sector's income matches its expenses. Then write the augmented matrix that can be row reduced to find these prices. The first, second, and third columns of the matrix should correspond to Chemicals, Fuels, and Machinery, respectively.

The augmented matrix is _____.
(Type an integer or decimal for each matrix element.)

c. Find a set of equilibrium prices when the price for the Machinery output is 40 units.

$p_{Chemicals}$ = _____, p_{Fuels} = _____, $p_{Machinery}$ = _____
(Type integers or decimals rounded to the nearest tenth as needed.)

ID: 1.6.3

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4. Suppose an economy has four sectors: Mining, Lumber, Energy, and Transportation. Mining sells 10% of its output to Lumber, 60% to Energy, and retains the rest. Lumber sells 15% of its output to Mining, 60% to Energy, 20% to Transportation, and retains the rest. Energy sells 30% of its output to Mining, 20% to Lumber, 25% to Transportation, and retains the rest. Transportation sells 10% of its output to Mining, 15% to Lumber, 50% to Energy, and retains the rest.
a. Construct the exchange table for this economy.
b. Find a set of equilibrium prices for this economy.

a. Complete the exchange table below.

Distribution of Output from:				
Mining	Lumber	Energy	Transportation	Purchased by:
_____	_____	_____	_____	Mining
_____	_____	_____	_____	Lumber
_____	_____	_____	_____	Energy
_____	_____	_____	_____	Transportation

(Type integers or decimals.)

b. Denote the prices (that is, dollar values) of the total annual outputs of the Mining, Lumber, Energy, and Transportation sectors by p_M, p_L, p_E, and p_T, respectively.

If p_T = \$100, then p_M = \$_____, p_L = \$_____, and p_E = \$_____.
(Round to the nearest dollar as needed.)

ID: 1.6.4

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5. Balance the chemical equation.

$B_2S_3 + H_2O \rightarrow H_3BO_3 + H_2S$

Assume the coefficient of H_2S is 3. What is the balanced equation?

_____ B_2S_3 + _____ $H_2O \rightarrow$ _____ H_3BO_3 + _____ H_2S

ID: 1.6.5

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6. Balance the following chemical equation.

$H_3O + CaCO_3 \rightarrow H_2O + Ca + CO_2$

Assume the coefficient of CO_2 is 1. What is the balanced equation?

_____ H_3O + _____ $CaCO_3 \rightarrow$ _____ H_2O + _____ $Ca + CO_2$

ID: 1.6.7

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7. Balance the chemical equation.

$PbN_6 + CrMn_2O_8 \rightarrow Pb_3O_4 + Cr_2O_3 + MnO_2 + NO$

Assume the coefficient of NO is 90. What is the balanced equation?

_____ PbN_6 + _____ $CrMn_2O_8 \rightarrow$ _____ Pb_3O_4 + _____ Cr_2O_3 + _____ Mn

ID: 1.6.9

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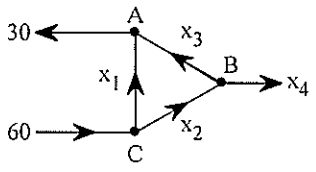
8. Find the general flow pattern of the network shown in the figure. Assuming that the flows are all nonnegative, what is the largest possible value for x_3?



Find the general flow pattern of the network shown in the figure. Choose the correct answer below and fill in the answer boxes to complete your choice.

○ **A.** $x_1 =$ _____ $x_2 =$ _____ $x_3 =$ _____ $x_4 =$ _____

○ **B.** x_1 is free $x_2 =$ _____ x_3 is free $x_4 =$ _____

○ **C.** $x_1 =$ _____ x_2 is free x_3 is free x_4 is free

○ **D.** $x_1 =$ _____ $x_2 =$ _____ x_3 is free $x_4 =$ _____

Assuming that the flows are all nonnegative, what is the largest possible value for x_3?

The largest possible value for x_3 is _____.

ID: 1.6.11

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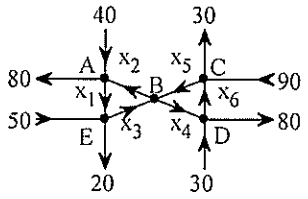
9. a. Find the general flow pattern of the network shown in the figure.

b. Assuming that the flow must be in the directions indicated, find the minimum flows in the branches denoted by x_2, x_3, x_4, and x_5.



a. Choose the correct answer below and fill in the answer boxes to complete your choice.

○ **A.**

$x_1 =$ _____
x_2 is free
$x_3 =$ _____
x_4 is free
x_5 is free
$x_6 =$ _____

○ **B.**

x_1 is free
$x_2 =$ _____
$x_3 =$ _____
$x_4 =$ _____
$x_5 =$ _____
$x_6 =$ _____

○ **C.**

$x_1 =$ _____
$x_2 =$ _____
x_3 is free
$x_4 =$ _____
$x_5 =$ _____
x_6 is free

○ **D.**

$x_1 =$ _____
x_2 is free
x_3 is free
x_4 is free
x_5 is free
$x_6 =$ _____

b. The minimum flow of x_2 is _____.

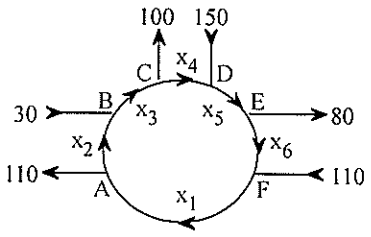
The minimum flow of x_3 is _____.

The minimum flow of x_4 is _____.

The minimum flow of x_5 is _____.

ID: 1.6.13

10. Intersections in England are often constructed as one-way "roundabouts," such as the one shown in the figure. Assume that traffic must travel in the directions shown. Find the general solution of the network flow. Find the smallest possible value for x_6.



What is the general solution of the network flow? Choose the correct answer below and fill in the answer boxes to complete your choice.

○ **A.**

x_1 is free
$x_2 =$ _____
$x_3 =$ _____
$x_4 =$ _____
$x_5 =$ _____
x_6 is free

○ **B.**

$x_1 =$ _____
$x_2 =$ _____
$x_3 =$ _____
$x_4 =$ _____
$x_5 =$ _____
x_6 is free

○ **C.**

x_1 is free
$x_2 =$ _____
x_3 is free
x_4 is free
$x_5 =$ _____
x_6 is free

○ **D.**

$x_1 =$ _____
x_2 is free
$x_3 =$ _____
$x_4 =$ _____
x_5 is free
x_6 is free

Find the smallest possible value for x_6.

The smallest possible value for x_6 is _____.

ID: 1.6.14